

Accelerated Fractional Programming for Multi-User ISAC Beamforming Optimization With Per-Group Power Constraints

Dogon Kim^{ID}, *Graduate Student Member, IEEE*, and Seok-Hwan Park^{ID}, *Senior Member, IEEE*

Abstract—This letter presents an efficient beamforming solution for a multi-user integrated sensing and communication (ISAC) system. The problem of optimizing ISAC beamforming vectors is formulated to maximize a weighted sum of communication sum-rate and radar sensing signal-to-interference-plus-noise ratio (SINR), subject to arbitrary per-group power constraints (PGPCs). To address the non-convexity, we employ fractional programming (FP) approach, transforming the original problem into a series of convex subproblems with fixed auxiliary variables. To avoid dependence on convex optimization tools, we propose an accelerated FP (A-FP) scheme that partitions the beamforming vectors into block variables, which are sequentially optimized with closed-form solutions for each block. Numerical results demonstrate that the A-FP scheme achieves almost identical performance to the FP scheme, while significantly reducing computational complexity.

Index Terms—Integrated sensing and communication, multi-user, beamforming, fractional programming.

I. INTRODUCTION

INTEGRATED sensing and communication (ISAC), where communication and sensing systems share both spectrum resources and hardware platforms, is envisioned as a key enabler for sixth-generation (6G) wireless systems [1], [2], [3], [4], [5], [6], [7]. Extensive research efforts have focused on addressing ISAC design challenges, including ISAC beamforming optimization. Unlike traditional communication-oriented beamforming, which primarily focused on maximizing data rates, ISAC beamforming aims to optimize beamforming vectors to jointly enhance the communication and radar sensing performance. For radar sensing, various sensing metrics have been considered in the literature, including beampattern matching error [2], [7], Cramér-Rao bound (CRB) for target parameter estimation [3], and target sensing signal-to-interference-plus-noise ratio (SINR), also referred to as radar SINR or signal-to-clutter-plus-noise ratio (SCNR) [4], [5], [6], [7].

A key challenge in designing ISAC beamforming in practical systems is the high computational complexity of optimization algorithms. Most existing works [2], [3], [4], [5], [6] derive optimal or locally optimal solutions by converting non-convex problems into sequences of convex problems, which are then solved using optimization tools such as CVX [8]. A recent work [7] proposed an efficient ISAC beamforming method based

on generalized power iteration (GPI), which updates beamforming vectors by solving eigenproblems instead of relying on optimization solvers. However, the GPI approach primarily focuses on sum-power constraint (SPC), and extending it to optimize ISAC beamforming under arbitrary per-group power constraints (PGPCs) is challenging due to the complexity of handling eigenproblems in this context.

In practical systems, base station (BS) antennas are often powered by separate amplifiers [9] or grouped into sectors [10] to enhance operational efficiency. PGPCs naturally arise in such configurations, encompassing SPC and per-antenna power constraints (PAPCs) as special cases. Additionally, PGPCs are relevant in distributed antenna systems such as cell-free massive multiple-input multiple-output (MIMO), where radio frequency (RF) power sharing across transmission points is infeasible. Beamforming optimization under PGPCs has been studied for communication systems [11], [12], [13], but its application to ISAC remains an open challenge.

In this letter, we develop an efficient optimization algorithm for multi-user ISAC beamforming with arbitrary PGPCs. We formulate the problem of maximizing a weighted sum of communication sum-rate and radar sensing SINR, subject to PGPCs. To address the non-convexity of the problem, we employ the fractional programming (FP) approach [14], which transforms fractional functions into a linear combination of affine and quadratic functions using auxiliary variables. With fixed auxiliary variables, the transformed problem becomes convex, enabling alternate optimization of beamforming vectors and auxiliary variables to achieve a locally optimal solution. However, solving for beamforming vectors requires optimization solvers, leading to high complexity.

To address this complexity, we propose an accelerated implementation of the FP algorithm, inspired by the reduced weighted mean squared error (R-WMMSE) approach [13], [15]. While R-WMMSE was designed to reduce the complexity of conventional WMMSE algorithms in communication systems, our accelerated FP (A-FP) algorithm is tailored for the more general ISAC beamforming problem. We partition the beamforming weights into block variables based on antenna group indices associated with PGPCs. These block variables are sequentially optimized, rather than jointly, with each solution obtained in closed form using Lagrangian duality. Numerical results validate that the proposed A-FP achieves performance nearly identical to the FP method while significantly reducing computational complexity.

The main contributions are summarized as follows:

- We propose a novel optimization framework for ISAC beamforming under practical PGPCs.
- We develop an accelerated FP algorithm that reduces computational complexity while achieving near-optimal performance compared to conventional FP methods.

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The authors are with the Division of Electronic Engineering, Jeonbuk National University, Jeonju 54896, South Korea (e-mail: kdg8728@jbnu.ac.kr; seokhwan@jbnu.ac.kr).

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- We validate the proposed algorithm through numerical simulations, demonstrating its effectiveness in optimizing ISAC beamforming with PGPCs.

II. SYSTEM MODEL

We consider an ISAC system, in which a BS equipped with N_T transmit antennas simultaneously sends independent messages to K single-antenna user equipments (UEs) and senses a nearby point-like target by processing a reflected echo signal received by N_R receive antennas.

A. Signal and Channel Model

Letting $\mathbf{x} \in \mathbb{C}^{N_T}$ denote the signal vector transmitted by the BS, the received signal y_k at communication UE $k \in \mathcal{K} \triangleq \{1, 2, \dots, K\}$ is expressed as

$$y_k^C = \mathbf{h}_k^H \mathbf{x} + z_k^C, \quad (1)$$

where $\mathbf{h}_k \in \mathbb{C}^{N_T}$ is the channel vector from the BS to UE k , and $z_k \sim \mathcal{CN}(0, \sigma_C^2)$ represents the additive noise.

The BS receives an echo signal vector $\mathbf{y}^S \in \mathbb{C}^{N_R}$, which is reflected by the environment, including the target and clutters. The observed echo signal $\mathbf{y}^S$ is modeled as

$$\mathbf{y}^S = \mathbf{G}_0 \mathbf{x} + \sum_{m \in \mathcal{M}} \mathbf{G}_m \mathbf{x} + \mathbf{G}_{\text{SI}} \mathbf{x} + \mathbf{z}^S, \quad (2)$$

where $\mathbf{G}_0 \in \mathbb{C}^{N_R \times N_T}$ is the target response matrix, $\mathbf{G}_m \in \mathbb{C}^{N_R \times N_T}$ for $m \in \mathcal{M} \triangleq \{1, 2, \dots, M\}$ are the target-free clutter channel matrices, $\mathbf{G}_{\text{SI}} \in \mathbb{C}^{N_R \times N_T}$ is the self-interference (SI) channel matrix, and $\mathbf{z}^S \sim \mathcal{CN}(\mathbf{0}, \sigma_S^2 \mathbf{I})$ is the additive noise vector. Each reflection matrix $\mathbf{G}_m, m \in \{0\} \cup \mathcal{M}$, is given by $\mathbf{G}_m = \alpha_m \mathbf{a}_R(\theta_m)(\mathbf{a}_T(\theta_m))^H$ [7], where $\alpha_m \sim \mathcal{CN}(0, \sigma_{\alpha, m}^2)$ is the complex reflection coefficient accounting for pathloss and radar cross-section, θ_m is the azimuth angle of arrival (AoA) of the target ($m = 0$) or the m th scatterer ($m \in \mathcal{M} \triangleq \{1, 2, \dots, M\}$), and $\mathbf{a}_R(\cdot) \in \mathbb{C}^{N_R}$ and $\mathbf{a}_T(\cdot) \in \mathbb{C}^{N_T}$ are the receive and transmit steering vectors, respectively. Assuming that the BS employs uniform linear arrays (ULAs) with half-wavelength spacing for both the transmitter and receiver, $\mathbf{a}_X(\theta)$ for $X \in \{T, R\}$ is given by $\mathbf{a}_X(\theta) = (1/\sqrt{N_X})[1, e^{j\pi \sin(\theta)}, \dots, e^{j\pi(N_X-1)\sin(\theta)}]^T$. We focus on the data transmission and radar tracking phase, which operates after an initial beam sweeping phase. Accordingly, we assume that the reflection coefficients $\{\alpha_m\}_{m=0}^M$ and the AoAs $\{\theta_m\}_{m=0}^M$ have been estimated through the beam sweeping process and are known at the BS [4], [6], [7].

The transmitted signal vector $\mathbf{x}$ is generated through ISAC beamforming, expressed as

$$\mathbf{x} = \sum_{k \in \mathcal{K}} \mathbf{v}_k^C s_k^C + \sum_{l \in \mathcal{L}} \mathbf{v}_l^S s_l^S, \quad (3)$$

where $s_k^C \in \mathbb{C}$ represents the data symbol intended for UE k , $s_l^S \in \mathbb{C}$ denotes the l th signal stream dedicated to radar sensing, and $\mathbf{v}_k^C \in \mathbb{C}^{N_T}$ and $\mathbf{v}_l^S \in \mathbb{C}^{N_T}$ are the corresponding beamforming vectors for s_k^C and s_l^S , respectively. It is assumed that both s_k^C and s_l^S are independent zero-mean and unit-variance signals. The set of radar sensing stream indices is denoted by $\mathcal{L} \triangleq \{1, 2, \dots, L\}$ with L being the number of radar sensing streams.

We impose general PGPCs on the transmitted signal vector $\mathbf{x}$. Specifically, we partition the set of transmit antenna

indices $\mathcal{N}_T \triangleq \{1, 2, \dots, N_T\}$ into G antenna groups $\mathcal{N}_{T,1}, \dots, \mathcal{N}_{T,G}$ such that $\cup_{g=1}^G \mathcal{N}_{T,g} = \mathcal{N}_T$ and $\mathcal{N}_{T,g_1} \cap \mathcal{N}_{T,g_2} = \emptyset, \forall g_1 \neq g_2$. Notably, the described PGPCs include SPC and PAPCs as special cases, since the SPC corresponds to $G = 1$ with $\mathcal{N}_{T,1} = \mathcal{N}_T$, while the PAPCs are modeled by $G = N_T$ with $\mathcal{N}_{T,i} = \{i\}, \forall i \in \mathcal{N}_T$. Under the defined antenna groups, the PGPCs are stated as

$$\sum_{k \in \mathcal{K}} \|\mathbf{E}_g^H \mathbf{v}_k^C\|^2 + \sum_{l \in \mathcal{L}} \|\mathbf{E}_g^H \mathbf{v}_l^S\|^2 \leq P_g, \quad \forall g \in \mathcal{G}, \quad (4)$$

where $\mathcal{G} \triangleq \{1, 2, \dots, G\}$ denotes the set of antenna group indices, P_g is the power budget for the g th antenna group, and $\mathbf{E}_g \in \mathbb{C}^{N_T \times N_{T,g}}$ is a submatrix of the identity matrix $\mathbf{I}_{N_T}$ that takes only the i th columns for the g th antenna group, i.e., $i \in \mathcal{N}_{T,g}$, where $N_{T,g} = |\mathcal{N}_{T,g}|$.

B. Communication and Sensing Performance Metrics

For given beamforming vectors $\mathbf{v} \triangleq \{\mathbf{v}_k^C\}_{k \in \mathcal{K}} \cup \{\mathbf{v}_l^S\}_{l \in \mathcal{L}}$, the data rate R_k^C for each UE k is given by

$$R_k^C = f_k^C(\mathbf{v}) = \log_2 \left(1 + \frac{|\mathbf{h}_k^H \mathbf{v}_k^C|^2}{\text{IF}_k^C(\mathbf{v}) + \sigma_C^2} \right), \quad (5)$$

where we have defined the interference power $\text{IF}_k^C(\mathbf{v}) = \sum_{k' \in \mathcal{K} \setminus \{k\}} |\mathbf{h}_k^H \mathbf{v}_{k'}^C|^2 + \sum_{l \in \mathcal{L}} |\mathbf{h}_k^H \mathbf{v}_l^S|^2$ encountered when decoding the signal s_k^C at UE k .

In the presence of clutter signals, the *sensing SINR*, also referred to as SCNR, serves as a widely used performance metric for radar sensing [4], [5], [6], [7]. It is defined as

$$\begin{aligned} \text{SINR}^S(\mathbf{v}) &= \frac{\mathbb{E}[\|\mathbf{G}_0 \mathbf{x}\|^2]}{\mathbb{E}[\|\mathbf{G}_{\text{cl}} \mathbf{x}\|^2] + \mathbb{E}[\|\mathbf{z}^S\|^2]} = \frac{\|\bar{\mathbf{G}}_0 \bar{\mathbf{v}}\|^2}{\|\bar{\mathbf{G}}_{\text{cl}} \bar{\mathbf{v}}\|^2 + \sigma_S^2 N_R} \\ &= \bar{\mathbf{v}}^H \bar{\mathbf{G}}_0^H \left[(\|\bar{\mathbf{G}}_{\text{cl}} \bar{\mathbf{v}}\|^2 + \sigma_S^2 N_R) \mathbf{I}_{(K+L)N_R} \right]^{-1} \bar{\mathbf{G}}_0 \bar{\mathbf{v}}, \end{aligned} \quad (6)$$

where $\bar{\mathbf{v}} = [(\mathbf{v}_1^C)^H \dots (\mathbf{v}_K^C)^H (\mathbf{v}_1^S)^H \dots (\mathbf{v}_L^S)^H]^H \in \mathbb{C}^{(K+L)N_T}$, $\bar{\mathbf{G}}_0 = \mathbf{I}_{K+L} \otimes \mathbf{G}_0 \in \mathbb{C}^{(L+K)N_R \times (L+K)N_T}$, and $\bar{\mathbf{G}}_{\text{cl}} = \mathbf{I}_{K+L} \otimes \mathbf{G}_{\text{cl}} \in \mathbb{C}^{(L+K)N_R \times (L+K)N_T}$ with $\mathbf{G}_{\text{cl}} = \sum_{m \in \mathcal{M}} \mathbf{G}_m + \mathbf{G}_{\text{SI}}$ denoting the clutter-plus-SI channel matrix.

III. FP-BASED ISAC BEAMFORMING OPTIMIZATION

In this section, we address the optimization of the ISAC beamforming vectors $\mathbf{v}$ by employing the FP approach introduced in [14]. As the ISAC beamforming simultaneously concerns two distinct objectives for communication and sensing, we define the objective function as a weighted sum¹ of the communication sum-rate $\sum_{k \in \mathcal{K}} f_k^C(\mathbf{v})$ and the sensing SINR defined in (6). Accordingly, the ISAC beamforming optimization problem is formulated as

$$\begin{aligned} \max_{\mathbf{v}} \quad & \rho_C \sum_{k \in \mathcal{K}} f_k^C(\mathbf{v}) + (1 - \rho_C) \text{SINR}^S(\mathbf{v}) \\ \text{s.t.} \quad & \sum_{k \in \mathcal{K}} \|\mathbf{E}_g^H \mathbf{v}_k^C\|^2 + \sum_{l \in \mathcal{L}} \|\mathbf{E}_g^H \mathbf{v}_l^S\|^2 \leq P_g, \quad \forall g \in \mathcal{G}, \end{aligned} \quad (7)$$

where $\rho_C \in [0, 1]$ is a predetermined weighting factor that controls the tradeoff between communication and sensing performance achieved by the solution of (7). A higher ρ_C

¹We can instead consider maximizing the sensing SINR while satisfying minimum data rate constraints. The FP approach applies directly to this problem, but extending the A-FP scheme in Section IV is not straightforward.

Algorithm 1 FP Algorithm for Problem (7)**1 initialize:**

2 Set $\mathbf{v}$ to arbitrary beamforming vectors that satisfy the PGPCs (4) and calculate the objective value $f_{\text{obj}}^{\text{old}} \leftarrow \rho_C \sum_{k \in \mathcal{K}} f_k^C(\mathbf{v}) + (1 - \rho_C) \text{SINR}^S(\mathbf{v})$ with the initialized $\mathbf{v}$.

3 repeat

4 Update the auxiliary variables $\boldsymbol{\gamma}$ and $\boldsymbol{\lambda}$ according to (8).

5 Update the beamforming vectors $\mathbf{v}$ as a solution of the problem (9) for fixed $\boldsymbol{\gamma}$ and $\boldsymbol{\lambda}$, and calculate the objective value $f_{\text{obj}}^{\text{new}} \leftarrow \rho_C \sum_{k \in \mathcal{K}} f_k^C(\mathbf{v}) + (1 - \rho_C) \text{SINR}^S(\mathbf{v})$ with the updated $\mathbf{v}$.

6 until $|f_{\text{obj}}^{\text{new}} - f_{\text{obj}}^{\text{old}}| \leq \delta \cdot f_{\text{obj}}^{\text{old}}$ (Otherwise, set $f_{\text{obj}}^{\text{old}} \leftarrow f_{\text{obj}}^{\text{new}}$)

prioritizes the maximization of the communication sum-rate, potentially at a sacrifice of the sensing SINR.

The objective function of (7) is non-convex due to its fractional nature. Fortunately, however, it has a structure amenable to the FP approach [14]. Utilizing the matrix quadratic transform [14, Th. 1] and the matrix Lagrangian dual transform [14, Th. 2], we derive lower bounds on each term $f_k^C(\mathbf{v})$ and $\text{SINR}^S(\mathbf{v})$ in the objective function as

$$f_k^C(\mathbf{v}) \geq \tilde{f}_k^C(\mathbf{v}, \gamma_k^C, \lambda_k^C), \text{ and } \text{SINR}^S(\mathbf{v}) \geq \tilde{f}^S(\mathbf{v}, \boldsymbol{\lambda}^S),$$

where $\gamma_k^C \geq 0$, $\lambda_k^C \in \mathbb{C}$ and $\boldsymbol{\lambda}^S \in \mathbb{C}^{(K+L)N_R}$ are auxiliary variables, and the functions $\tilde{f}_k^C(\mathbf{v}, \gamma_k^C, \lambda_k^C)$ and $\tilde{f}^S(\mathbf{v}, \boldsymbol{\lambda}^S)$ are defined as

$$\begin{aligned} \tilde{f}_k^C(\mathbf{v}, \gamma_k^C, \lambda_k^C) &= \log_2 \left(1 + \gamma_k^C \right) - \frac{\gamma_k^C}{\ln 2} + \frac{1 + \gamma_k^C}{\ln 2} \times \\ &\quad \left(2 \text{Re} \left\{ (\mathbf{v}_k^C)^H \mathbf{h}_k \lambda_k^C \right\} - |\lambda_k^C|^2 \left(|\mathbf{h}_k^H \mathbf{v}_k^C|^2 + \text{IF}_k^C(\mathbf{v}) + \sigma_C^2 \right) \right), \\ \tilde{f}^S(\mathbf{v}, \boldsymbol{\lambda}^S) &= 2 \text{Re} \left\{ \bar{\mathbf{v}}^H \bar{\mathbf{G}}_0^H \boldsymbol{\lambda}^S \right\} - \|\boldsymbol{\lambda}^S\|^2 \left(\|\bar{\mathbf{G}}_{\text{cl}} \bar{\mathbf{v}}\|^2 + \sigma_S^2 N_R \right). \end{aligned}$$

These lower bounds become tight when the auxiliary variables γ_k^C , λ_k^C and $\boldsymbol{\lambda}^S$ are optimally chosen as

$$\gamma_k^C = \left| \mathbf{h}_k^H \mathbf{v}_k^C \right|^2 / \left(\text{IF}_k^C(\mathbf{v}) + \sigma_C^2 \right), \quad (8a)$$

$$\lambda_k^C = \mathbf{h}_k^H \mathbf{v}_k^C / \left(\left| \mathbf{h}_k^H \mathbf{v}_k^C \right|^2 + \text{IF}_k^C(\mathbf{v}) + \sigma_C^2 \right), \quad (8b)$$

$$\boldsymbol{\lambda}^S = \bar{\mathbf{G}}_0 \bar{\mathbf{v}} / \left(\|\bar{\mathbf{G}}_{\text{cl}} \bar{\mathbf{v}}\|^2 + \sigma_S^2 N_R \right). \quad (8c)$$

Leveraging these lower bounds, we can equivalently reformulate the original problem (7) as

$$\begin{aligned} \max_{\mathbf{v}, \boldsymbol{\gamma}, \boldsymbol{\lambda}} \quad & \rho_C \sum_{k \in \mathcal{K}} \tilde{f}_k^C(\mathbf{v}, \gamma_k^C, \lambda_k^C) + (1 - \rho_C) \tilde{f}^S(\mathbf{v}, \boldsymbol{\lambda}^S) \quad (9) \\ \text{s.t.} \quad & \sum_{k \in \mathcal{K}} \left\| \mathbf{E}_g^H \mathbf{v}_k^C \right\|^2 + \sum_{l \in \mathcal{L}} \left\| \mathbf{E}_g^H \mathbf{v}_l^S \right\|^2 \leq P_g, \quad \forall g \in \mathcal{G}, \end{aligned}$$

where $\boldsymbol{\gamma} = \{\gamma_k^C\}_{k \in \mathcal{K}}$ and $\boldsymbol{\lambda} = \{\lambda_k^C\}_{k \in \mathcal{K}} \cup \{\boldsymbol{\lambda}^S\}$ denote the sets of auxiliary variables.

Although the problem (9) still remains non-convex, it becomes convex with respect to $\mathbf{v}$ when the auxiliary variables $\boldsymbol{\gamma}$ and $\boldsymbol{\lambda}$ are fixed. Moreover, the optimal values of $\boldsymbol{\gamma}$ and $\boldsymbol{\lambda}$ for a given $\mathbf{v}$ can be directly obtained from (8). This observation allows us to derive an iterative algorithm that alternates between updating the primal beamforming variables $\mathbf{v}$ and the auxiliary variables $\boldsymbol{\gamma}$ and $\boldsymbol{\lambda}$. This iterative process guarantees monotonically non-decreasing objective values, ensuring convergence to a local optimum. The detailed steps of the iterative algorithm are summarized in Algorithm 1. When updating the beamforming vectors $\mathbf{v}$ at Step 5 of each iteration, the convex problem can be solved by using standard convex optimization tools such as CVX [8].

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Remark 1: We have applied both the matrix quadratic transform [14, Th. 1] and the matrix Lagrangian dual transform [14, Th. 2] to derive the problem (9). If sensing is not considered (i.e., $\rho_C = 1$), only the latter transform [14, Th. 2] is required.

IV. ACCELERATING FP-BASED OPTIMIZATION

A key challenge in the FP-based optimization in Section III is its high computational complexity, primarily due to the convex optimization required in Step 5 of Algorithm 1. To mitigate this, we propose an accelerated version, the A-FP scheme, which replaces the convex optimization step with sequential closed-form updates for each antenna group.

A. Accelerated FP With Per-Group Closed-Form Updates

To accelerate the beamforming update process, we draw inspiration from the R-WMMSE approach [13], [15], which was developed aiming to reduce the complexity inherent in conventional WMMSE algorithms. Unlike existing R-WMMSE approaches, which focused primarily on communication-oriented beamforming problems, our A-FP algorithm is designed for more general ISAC beamforming optimization problem.

To elaborate, we define a vector $\mathbf{v}_{A,g} \in \mathbb{C}^{(K+L)N_{T,g}}$ that aggregates the beamforming vectors corresponding to the g th antenna group as

$$\mathbf{v}_{A,g} = \left[(\mathbf{v}_{1,g}^C)^H \cdots (\mathbf{v}_{K,g}^C)^H (\mathbf{v}_{1,g}^S)^H \cdots (\mathbf{v}_{L,g}^S)^H \right]^H, \quad (10)$$

where $\mathbf{v}_{k,g}^C \in \mathbb{C}^{N_{T,g}}$ and $\mathbf{v}_{l,g}^S \in \mathbb{C}^{N_{T,g}}$ are subvectors of $\mathbf{v}_k^C$ and $\mathbf{v}_l^S$, respectively, corresponding to the antenna indices in $\mathcal{N}_{T,g}$.

The key difference in the A-FP scheme lies in Step 5 of Algorithm 1. Instead of jointly optimizing all beamforming vectors $\mathbf{v}$ for fixed auxiliary variables $\boldsymbol{\gamma}$ and $\boldsymbol{\lambda}$, we alternately optimize the G block variables $\mathbf{v}_{A,1}, \mathbf{v}_{A,2}, \dots, \mathbf{v}_{A,G}$. Specifically, we seek closed-form solution for optimizing each block variable $\mathbf{v}_g$ for fixed others. To achieve this, we express the objective function in the problem (9) with respect to $\mathbf{v}_{A,g}$ as

$$\begin{aligned} & \rho_C \sum_{k \in \mathcal{K}} \tilde{f}_k^C(\mathbf{v}, \gamma_k^C, \lambda_k^C) + (1 - \rho_C) \tilde{f}^S(\mathbf{v}, \boldsymbol{\lambda}^S) \\ &= -\mathbf{v}_{A,g}^H \mathbf{Q}_g \mathbf{v}_{A,g} - 2 \text{Re} \left\{ \mathbf{b}_g^H \mathbf{v}_{A,g} \right\} + c_g, \end{aligned} \quad (11)$$

where c_g is a constant not affected by $\mathbf{v}_{A,g}$, and $\mathbf{Q}_g \in \mathbb{C}^{(K+L)N_{T,g} \times (K+L)N_{T,g}}$ and $\mathbf{b}_g \in \mathbb{C}^{(K+L)N_{T,g}}$ are defined as

$$\begin{aligned} \mathbf{Q}_g &= \frac{\rho_C}{\ln 2} \sum_{k \in \mathcal{K}} \left(1 + \gamma_k^C \right) |\lambda_k^C|^2 \mathbf{Q}_{g,k} + (1 - \rho_C) \|\boldsymbol{\lambda}^S\|^2 \mathbf{Q}_g^S, \\ \mathbf{b}_g &= \frac{\rho_C}{\ln 2} \sum_{k \in \mathcal{K}} \left(1 + \gamma_k^C \right) \left[|\lambda_k^C|^2 \mathbf{b}_{g,k}^C - \left(\lambda_k^C \right) \mathbf{E}_{g,k}^C \mathbf{h}_{k,g} \right] \\ &\quad + (1 - \rho_C) \left[\|\boldsymbol{\lambda}^S\|^2 \mathbf{b}_g^S - \left(\mathbf{I}_{K+L} \otimes \bar{\mathbf{G}}_{0,g}^H \right) \boldsymbol{\lambda}^S \right], \end{aligned}$$

with $\mathbf{Q}_{g,k}^C, \mathbf{Q}_g^S \in \mathbb{C}^{(K+L)N_{T,g} \times (K+L)N_{T,g}}$, and $\mathbf{b}_{g,k}^C, \mathbf{b}_g^S \in \mathbb{C}^{(K+L)N_{T,g}}$ given by

$$\begin{aligned} \mathbf{Q}_{g,k}^C &= \sum_{k' \in \mathcal{K}} \mathbf{E}_{g,k'}^C \mathbf{h}_{k',g} \mathbf{h}_{k,g}^H \left(\mathbf{E}_{g,k'}^C \right)^H + \sum_{l \in \mathcal{L}} \mathbf{E}_{g,l}^S \mathbf{h}_{l,g} \mathbf{h}_{k,g}^H \left(\mathbf{E}_{g,l}^S \right)^H, \\ \mathbf{Q}_g^S &= \sum_{k \in \mathcal{K}} \mathbf{E}_{g,k}^C \mathbf{G}_{\text{cl},g}^H \mathbf{G}_{\text{cl},g} \left(\mathbf{E}_{g,k}^C \right)^H + \sum_{l \in \mathcal{L}} \mathbf{E}_{g,l}^S \mathbf{G}_{\text{cl},g}^H \mathbf{G}_{\text{cl},g} \left(\mathbf{E}_{g,l}^S \right)^H, \end{aligned}$$

Algorithm 2 Accelerated FP (A-FP) Algorithm**1 Initialize:**

2 Set $\mathbf{v}$ to arbitrary beamforming vectors that satisfy the PGPCs (4) and calculate the objective value $f_{\text{obj}}^{\text{old}} \leftarrow \rho_C \sum_{k \in \mathcal{K}} f_k^C(\mathbf{v}) + (1 - \rho_C) \text{SINR}^S(\mathbf{v})$ with the initialized $\mathbf{v}$.

3 Repeat

4 Update the auxiliary variables $\boldsymbol{\gamma}$ and $\boldsymbol{\lambda}$ according to (8).

5 For $g \leftarrow 1$ to G

Update $\mathbf{v}_{A,g}$ according to (13) with the optimal λ_g^* found via the bisection method, and calculate the objective value $f_{\text{obj}}^{\text{new}} \leftarrow \rho_C \sum_{k \in \mathcal{K}} f_k^C(\mathbf{v}) + (1 - \rho_C) \text{SINR}^S(\mathbf{v})$ with the updated $\mathbf{v}$.

End

6 until $|f_{\text{obj}}^{\text{new}} - f_{\text{obj}}^{\text{old}}| \leq \delta \cdot f_{\text{obj}}^{\text{old}}$ (Otherwise, set $f_{\text{obj}}^{\text{old}} \leftarrow f_{\text{obj}}^{\text{new}}$)

$$\begin{aligned} \mathbf{b}_{g,k}^C &= \left(\sum_{k' \in \mathcal{K}} A_{g,k,k'}^C \mathbf{E}_{g,k'}^C + \sum_{l \in \mathcal{L}} A_{g,k,l}^S \mathbf{E}_{g,l}^S \right) \mathbf{h}_{k,g}, \\ \mathbf{b}_g^S &= \sum_{k \in \mathcal{K}} \mathbf{E}_{g,k}^C \mathbf{G}_{\text{cl},g}^H \sum_{g' \in \mathcal{G} \setminus \{g\}} \mathbf{G}_{\text{cl},g'}^H \mathbf{v}_{k,g'}^C \\ &\quad + \sum_{l \in \mathcal{L}} \mathbf{E}_{g,l}^S \mathbf{G}_{\text{cl},g}^H \sum_{g' \in \mathcal{G} \setminus \{g\}} \mathbf{G}_{\text{cl},g'}^H \mathbf{v}_{l,g'}^S. \end{aligned}$$

Here we have defined $A_{g,k,k'}^C = \sum_{g' \in \mathcal{G} \setminus \{g\}} \mathbf{h}_{k,g'}^H \mathbf{v}_{k',g'}^C$ and $A_{g,k,l}^S = \sum_{g' \in \mathcal{G} \setminus \{g\}} \mathbf{h}_{k,g'}^H \mathbf{v}_{l,g'}^S$ and the subvectors and submatrices corresponding to the g th antenna group $\mathcal{N}_{T,g}$ as $\mathbf{h}_{k,g} = \mathbf{E}_g^H \mathbf{h}_k \in \mathbb{C}^{N_{T,g}}$, $\mathbf{v}_{k,g}^C = \mathbf{E}_g^H \mathbf{v}_k^C \in \mathbb{C}^{N_{T,g}}$, $\mathbf{v}_{l,g}^S = \mathbf{E}_g^H \mathbf{v}_l^S \in \mathbb{C}^{N_{T,g}}$, $\mathbf{G}_{0,g} = \mathbf{G}_0 \mathbf{E}_g \in \mathbb{C}^{N_R \times N_{T,g}}$ and $\mathbf{G}_{\text{cl},g} = \mathbf{G}_{\text{cl}} \mathbf{E}_g \in \mathbb{C}^{N_R \times N_{T,g}}$. Also, $\mathbf{E}_{g,k}^C \in \mathbb{C}^{(K+L)N_{T,g} \times N_{T,g}}$ and $\mathbf{E}_{g,l}^S \in \mathbb{C}^{(K+L)N_{T,g} \times N_{T,g}}$ are defined as $\mathbf{E}_{g,k}^C = [\mathbf{0}_{(k-1)N_{T,g} \times N_{T,g}}^H, \mathbf{I}_{N_{T,g}}, \mathbf{0}_{(K-k+L)N_{T,g} \times N_{T,g}}^H]^H$ and $\mathbf{E}_{g,l}^S = [\mathbf{0}_{(K+l-1)N_{T,g} \times N_{T,g}}^H, \mathbf{I}_{N_{T,g}}, \mathbf{0}_{(L-l)N_{T,g} \times N_{T,g}}^H]^H$, respectively.

Consequently, the optimization of the beamforming weights $\mathbf{v}_{A,g}$ associated with the antenna group g for fixed other beamforming weights $\{\mathbf{v}_{A,g'}\}_{g' \in \mathcal{G} \setminus \{g\}}$ and auxiliary variables $\boldsymbol{\gamma}$ and $\boldsymbol{\lambda}$ can be formulated as

$$\min_{\mathbf{v}_{A,g}} \mathbf{v}_{A,g}^H \mathbf{Q}_g \mathbf{v}_{A,g} + 2 \text{Re} \{ \mathbf{b}_g^H \mathbf{v}_{A,g} \} \quad \text{s.t.} \quad \|\mathbf{v}_{A,g}\|^2 \leq P_g. \quad (12)$$

Since $\mathbf{Q}_g \succeq \mathbf{0}$, the problem (12) is a convex quadratic problem. By applying Lagrangian duality, the optimal solution is given in a closed form as

$$\mathbf{v}_{A,g} = -(\mathbf{Q}_g + \mu_g^* \mathbf{I}_{(K+L)N_{T,g}})^{-1} \mathbf{b}_g, \quad (13)$$

where the optimal Lagrange multiplier μ_g^* is given by the smallest value of $\mu_g \geq 0$ satisfying the constraint $\|(\mathbf{Q}_g + \mu_g \mathbf{I}_{(K+L)N_{T,g}})^{-1} \mathbf{b}_g\|^2 \leq P_g$. Such a value μ_g^* can be efficiently found using the bisection method, as $\|(\mathbf{Q}_g + \mu_g \mathbf{I}_{(K+L)N_{T,g}})^{-1} \mathbf{b}_g\|^2$ monotonically decreases with μ_g . The complete A-FP algorithm is described in Algorithm 2.

Remark 2: If multiple targets are involved, the sensing metric can be defined as the sum of sensing SINRs across all targets. The fractional structure of this metric remains intact, allowing the FP and A-FP algorithms to be readily extended to this scenario.

B. Complexity Analysis

The complexity of Algorithm 1, which relies on solving the convex problem in Step 5, is upper bounded by $\mathcal{O}(n_V(n_V^3 + n_O))$ [16, p. 4], where n_V is the number of optimization

variables, and n_O is the number of arithmetic operations required to compute the objective and constraint functions. Assuming $N_{T,g} = N_T/G$, $\forall g \in \mathcal{G}$, they are given by $n_V = \mathcal{O}((K+L)N_T)$ and $n_O = \mathcal{O}(K(K+L)^2 N_T N_R)$. This results in an overall complexity of $\mathcal{O}((K+L)^4 N_T^4)$ for Step 5, assuming $N_T = N_R$.

In contrast, the A-FP scheme in Algorithm 2 significantly reduces computational complexity by avoiding the need to solve a general convex problem in Step 5. Instead, it performs closed-form computations through per-group beamforming updates. Specifically, the complexity of updating $\mathbf{v}_{A,g}$ for each antenna group g is dominated by the matrix inversion $(\mathbf{Q}_g + \mu_g^* \mathbf{I}_{(K+L)N_{T,g}})^{-1}$. Utilizing the block-diagonal structure of the matrix, which consists of $(K+L)$ blocks each of size (N_T/G) , the complexity of this operation is given by $\mathcal{O}((K+L)N_T^3/G^3)$. Since there are G antenna groups, the overall complexity is $\mathcal{O}((K+L)N_T^3/G^2)$. Compared to the FP scheme, the A-FP scheme reduces the computational burden by lowering all the orders of K , L , and N_T .

V. NUMERICAL RESULTS

In this section, we assess the communication and sensing performance of the proposed FP and A-FP schemes. For the antenna groups, we set $N_{T,g} = N_T/G$ with $\mathcal{N}_{T,g} = \{(g-1)N_T/G + 1, \dots, gN_T/G\}$ for all $g \in \mathcal{G}$. We assume symmetric power budgets across antenna groups, i.e., $P_g = P_{\text{total}}/G$, $\forall g \in \mathcal{G}$, where P_{total} denotes the total transmit power budget of the ISAC BS. We adopt a Rician fading channel model for communication UEs, where each channel vector $\mathbf{h}_k$ is modeled as $\mathbf{h}_k = \sqrt{\kappa_C/(\kappa_C+1)} \mathbf{h}_k^{\text{LoS}} + \sqrt{1/(\kappa_C+1)} \mathbf{h}_k^{\text{NLoS}}$. Here $\kappa_C \geq 0$ represents the Rician factor, and $\mathbf{h}_k^{\text{LoS}} = \mathbf{a}_T(\theta_k^C)$ and $\mathbf{h}_k^{\text{NLoS}} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_T})$ denote the LoS and non-LoS components, respectively, with θ_k^C being the azimuth angle of departure (AoD) for UE k . Following [17], the SI channel is modeled as $\mathbf{G}_{\text{SI}} \sim \mathcal{CN}(\sqrt{\sigma_{\text{SI}}^2 \kappa_{\text{SI}}/(1+\kappa_{\text{SI}})} \bar{\mathbf{G}}_{\text{SI}}, \sigma_{\text{SI}}^2/(1+\kappa_{\text{SI}}) \mathbf{I}_{N_R N_T})$ with the Rician factor $\kappa_{\text{SI}} \geq 0$, a deterministic component $\bar{\mathbf{G}}_{\text{SI}}$ having all-one elements without loss of generality [17], and the residual SI power σ_{SI}^2 . The signal-to-noise ratio (SNR) of the communication channel is defined as $P_{\text{total}}/\sigma_C^2$. For the sensing channel, we define the SNR, SI-to-noise ratio (SI2NR) and the clutter-to-noise ratio (CNR) of the sensing channel as $P_{\text{total}}\sigma_{\alpha,0}^2/\sigma_S^2$, $P_{\text{total}}\sigma_{\text{SI}}^2/\sigma_S^2$, and $P_{\text{total}}\sigma_{\alpha,\text{cl}}^2/\sigma_S^2$, respectively, where $\sigma_{\alpha,m}^2 = \sigma_{\alpha,\text{cl}}^2$, $\forall m \in \mathcal{M}$. We assume the presence of a single clutter, i.e., $M = 1$. Two baseline schemes are considered: MRT-FP and ZF-FP. In these schemes, maximum-ratio transmission (MRT) or zero-forcing (ZF) is used for the communication beamformers, and Algorithm 1 can be adopted with slight modification to optimize both schemes.

Fig. 1 plots the average objective value versus the number of iterations for a multi-user ISAC system with $L = 10$, $K = 8$, $G = N_T/2$, 10 dB SNR for communication and sensing channels, 5 dB SI2NR and CNR, $\kappa_C = 10$ dB, $\kappa_{\text{SI}} = 0$ dB, and $\rho_C = 0.3$. The graph demonstrates that the objective values of both the FP and A-FP schemes converge within a few iterations across all simulated scenarios. Notably, the objective value of A-FP closely approaches that of FP at every iteration step.

In Fig. 2, we illustrate the tradeoff between the average sum-rate and the average sensing SINR for a multi-user ISAC system

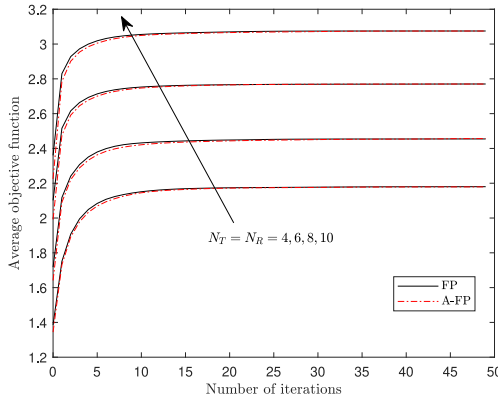


Fig. 1. Average objective value versus the number of iterations.

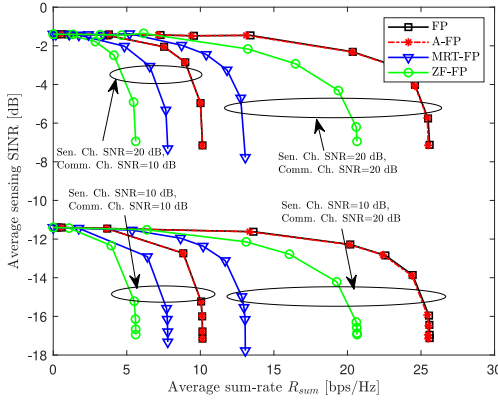


Fig. 2. Tradeoff between the sum-rate and sensing SINR.

TABLE I
AVERAGE ALGORITHM RUNTIME OF FP AND A-FP SCHEMES

$N_T = N_R = L$		10		20	
ρ_C		0.2	0.6	0.2	0.6
Avg. time [sec]	FP	145	141	153	154
	A-FP	1.8	1.9	4	4
Saving [%]		98.76	98.65	97.39	97.40

with $N_T = N_R = L = 10$, $K = 8$, $G = 5$, 5 dB SI2NR and CNR, $\kappa_C = 10$ dB, and $\kappa_{SI} = 0$ dB. Two different SNR levels are considered for the communication and sensing channels, and we assume that the clutter's and UEs' angles are uniformly distributed within $[-180^\circ, 180^\circ]$, while the target angle is fixed at $\theta_0 = 0^\circ$. Each curve is obtained by conducting simulations for varying values of $\rho_C \in [0, 1]$. It is noted that as ρ_C increases from 0, the sum-rate improves significantly with only a slight reduction in the sensing SINR. This occurs because, with a small ρ_C , the objective is still predominantly influenced by the sensing SINR, resulting in a beamforming solution that maintains strong sensing performance. However, due to the inclusion of the sum-rate metric in the objective, the sum-rate is notably improved compared to the case with $\rho_C = 0$, where the sum-rate is completely disregarded. Furthermore, the A-FP scheme achieves a tradeoff very close to the FP algorithm and significantly outperforms both the MRT-FP and ZF-FP schemes across all simulated scenarios.

Table I presents a comparison of the average algorithm runtimes of the FP and A-FP schemes under the same scenario as in Fig. 2, with the exception that $N_T = N_R = L \in \{10, 20\}$

and the SNR levels for communication and sensing channels are fixed at 10 dB. The A-FP scheme achieves complexity savings ranging from 97.39% to 98.76% across all simulated cases, with negligible performance loss as shown in Fig. 2. This highlights the efficiency of the A-FP scheme in terms of computational complexity.

VI. CONCLUSION

We have investigated the problem of optimizing multi-user ISAC beamforming vectors to maximize a weighted sum of communication sum-rate and sensing SINR, subject to arbitrary PGPCs. First, we employed the FP approach, which alternately optimizes the beamforming vectors and auxiliary variables using standard optimization tools. To improve computational efficiency, we proposed the A-FP scheme, which partitions the beamforming vectors into block variables and sequentially optimizes each block using closed-form solutions. Numerical results demonstrated that the A-FP scheme achieves up to 98.76% complexity reduction with negligible performance loss compared to the FP scheme.

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